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THE k-FOLD DIVISOR FUNCTION OVER THE INTERSECTION OF PIATETSKI-SHAPIRO SEQUENCES

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Abstract. The average of the k-fold divisor function

$$d_k(n) = \#\{(a_1, \ldots, a_k) \in \mathbb{Z}^k_{>0} : a_1 \cdots a_k = n\}$$

has been widely studied. The Piatetski–Shapiro sequences are of the form $\mathcal{N}^c = (\lfloor n^c \rfloor)_{n=1}^{\infty}$ with $c > 1,\ c \notin \mathbb{N}$. In this article, we estimate the average of the k-fold divisor function over the intersection of Piatetski-Shapiro sequences.

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1. Introduction

In number theory, the study of divisor functions occupies a central position. The classical divisor function, counting the number of positive divisors of an integer n, is given by $d(n) = \sum_{d|n} 1$. This notion admits a natural extension to the k-fold divisor function, which enumerates k-tuples of positive integers whose product equals n:

$$d_k(n) = \#\{(a_1, \ldots, a_k) \in \mathbb{N}^k : a_1 \cdots a_k = n\}.$$

Analytically, these functions are connected to the Riemann zeta function *via* the Dirichlet series identity

$$\sum_{n=1}^{\infty} \frac{d_k(n)}{n^s} = \zeta(s)^k, \qquad \Re(s) > 1.$$

When $k = 2$, one recovers the ordinary divisor function $d(n)$. A classical result states that

$$\sum_{n \leqslant x} d_k(n) = \big(1 + o(1)\big) x P_{k-1}(\log x),$$

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where $P_{k-1}(t)$ is a polynomial of degree $k-1$ in t, explicitly given by

$$P_{k-1}(\log x) = \operatorname*{Res}_{s=1} x^{s-1}\zeta^k(s)s^{-1}.$$

This paper focuses on averages of divisor functions over special sparse sequences. A sequence of particular interest in analytic number theory is the Piatetski-Shapiro sequence, parameterized by a real exponent $c > 1$ not an integer. It is defined by taking integer parts of fractional powers:

$$\mathcal{N}^c = \big(\lfloor n^c \rfloor\big)_{n=1}^{\infty},$$

where $\lfloor\cdot\rfloor$ denotes the floor function. When c is non-integral, the sequence $\mathcal{N}^c$ exhibits a sparse but structured distribution among the integers.

In his pioneering work, Piatetski-Shapiro [1] established an analogue of the prime number theorem for these sequences. He proved that for exponents c in the interval $(1, 12/11)$,

$$\#\{p \leqslant X : p \in \mathcal{N}^c\} = (1 + o(1))\frac{X^{1/c}}{\log X} \quad \text{as } X \to \infty. \tag{1.1}$$

This result can be viewed as an approximation to the famous unsolved problem concerning primes of the form $n^2 + 1$.

Subsequent research has substantially widened the admissible range for the exponent c. The strongest asymptotic result currently available, due to Rivat and Sargos [2], holds for $1 < c < 2817/2426$. When one relaxes the requirement to a lower bound (rather than an asymptotic formula), Rivat and Wu [3] obtained the larger interval $1 < c < 243/205$. A comprehensive survey of developments on primes in Piatetski-Shapiro sequences can be found in [4].

For the average of $d_k(n)$ over a single Piatetski-Shapiro sequence, Arkhipov, Saliba and Chubarikov [5] obtained an asymptotic formula valid for $1 < c < 8/7$. Their result states

$$\sum_{\substack{n \leqslant x\\ n \in \mathcal{N}^c}} d_k(n) = x^{1/c}\, Q_{k-1}(\log x) + O\left(\frac{x^{1/c}}{\log x}\right),$$

where Q_{k-1} is a polynomial of degree $k-1$. Lü and Zhai [6] later extended the admissible range to $c < 495/433$. In the special case $k = 2$, Wang and Zhang [7] proved the more precise asymptotic

$$\sum_{\substack{n \leqslant x\\ n \in \mathcal{N}^c}} d(n) = x^{\gamma}\log x + (2E - c)x^{\gamma} + O\left(\frac{x^{\gamma}}{\log x}\right) \qquad (1 < c < 6/5),$$

where E denotes Euler's constant and $\gamma = 1/c$.

A natural generalization is to consider primes (or divisor functions) over the intersection of several distinct Piatetski-Shapiro sequences. For a d-tuple $\boldsymbol{c} = (c_1, \dots, c_d)$ with each $c_i > 1$ not an integer and $c_i \neq c_j$ $(i \neq j)$, define

$$\mathcal{N}^{\boldsymbol{c}} = \{n : n = \lfloor n_1^{c_1} \rfloor = \cdots = \lfloor n_d^{c_d} \rfloor \text{ for some integers } n_1, \dots, n_d\}.$$

Set $\gamma_j = 1/c_j$, $\delta_j = 1 - \gamma_j$, and $\sigma = \delta_1 + \cdots + \delta_d$. One expects that for some σ_d such that $\sigma < \sigma_d$, it follows that

$$\#\{p \leqslant X : p \in \mathcal{N}^{\boldsymbol{c}}\} = (1 + o(1))\frac{\gamma_1 \cdots \gamma_d}{1 - \sigma}\frac{X^{1-\sigma}}{\log X}. \tag{1.2}$$

TABLE 1. Known ranges of σ guaranteeing the asymptotic (1.2).

Authors	Range of σ
Leitmann [8]	$\sigma < \frac{1}{28}$
Sirota [9]	$\sigma < \frac{1}{16}$
Zhai [10]	$\sigma < \frac{d}{4d^2+2}$ $(d \geqslant 3)$
Baker [11]	$\sigma < \frac{1}{12}$ $(d = 2, 3)$, $\sigma < \frac{1}{16}$ $(d = 4)$, $\sigma < \frac{1}{18}$ $(d = 5)$, $\sigma < \frac{1}{3d}$ $(d \geqslant 6)$
Guo, Guo and Jing [12]	$\sigma < \frac{290}{3297}$ $(2 \leqslant d \leqslant 10), \sigma < \frac{1}{d+1}$ $(d \geqslant 11)$

Previous works have established the asymptotic formula (1.2) under various conditions on the parameter σ. The following table summarizes the known admissible ranges; see Table 1.

Given the considerable attention devoted to the intersection of Piatetski-Shapiro sequences, it is natural to investigate the average of the k-fold divisor function over this set. Our main result is the following asymptotic formula.

Theorem 1.1. *With the notation above, assume that σ satisfies:*

- *For* $2 \leqslant k \leqslant 7$,

$$\sigma < \min\left(\frac{290}{3297}, \frac{1}{2k}, \frac{1}{d+1}\right).$$

- *For* $k \geqslant 8$,

$$\sigma < \min\left(\frac{290}{4137}, \frac{1}{d+1}\right).$$

Then we have

$$\sum_{\substack{n \leqslant x \\ n \in \mathcal{N}^c}} d_k(n) = (\gamma_1 \cdots \gamma_d)\big(1 + O((\log x)^{-1})\big)x^{1-\sigma} F_{k-1}(\log x),$$

where $F_{k-1}(\log x)$ *is a polynomial of degree* $k-1$*, explicitly given by*

$$F_{k-1}(\log x) = \operatorname*{Res}_{s=1-\sigma} x^{s-1}\zeta^k(s+\sigma)s^{-1}.$$

2. PRELIMINARIES

2.1. Notation

We denote by $\lfloor t \rfloor$ and $\{t\}$ the integral part and the fractional part of t, respectively. Let

$$\mathbf{e}(t) = \mathrm{e}^{2\pi i t} \qquad \text{and} \qquad \{t\} = t - \lfloor t \rfloor.$$

We use notation of the form $m \sim M$ as an abbreviation for $M < m \leqslant 2M$.

Throughout the paper, ε always denotes an arbitrarily small positive constant, which may not be the same at different occurrences. For given functions F and G, the notations $F \ll G$, $G \gg F$ and $F = O(G)$ are all equivalent to the statement that the inequality $|F| \leqslant \mathcal{C}|G|$ holds with some constant $\mathcal{C} > 0$.

2.2. The characterization of numbers in the intersection of Piatetski-Shapiro sequences

For a given real c with $1 < c < 2$, we denote by $\mathbf{1}_c(\cdot)$ the characteristic function of numbers in the Piatetski-Shapiro sequence $\mathcal{N}^c$, which is

$$\mathbf{1}_c(m) = \begin{cases} 1, & \text{if } m \in \mathcal{N}^c, \\ 0, & \text{if } m \notin \mathcal{N}^c. \end{cases}$$

Lemma 2.1. *Let f be an arithmetic function, and let $1 < c_1, \cdots, c_d < 2$ be real numbers and $c_i \neq c_j$ for all $i \neq j$. Denote $\gamma_j = 1/c_j$ and $\sigma_j = 1 - \gamma_j$ for $j = 1, \cdots, d$. Let $x > 0$ be a real number, $\mathcal{L} = \log x$, $1 < a < 1 + \mathcal{L}^{-1}$ and let*

$$\mathcal{M} = \mathcal{M}(c_1, \cdots, c_d, f, x) = \sum_{x < m \leqslant ax} f(m) \mathbf{1}_{c_1}(m) \cdots \mathbf{1}_{c_d}(m).$$

We have

$$\mathcal{M} = \gamma_1 \cdots \gamma_d x^{-(\delta_1 + \cdots + \delta_d)} \Bigg((1 + O(\mathcal{L}^{-1})) \sum_{x < m \leqslant ax} f(m) + O\Bigg(\sum_{\substack{\boldsymbol{h} \in \mathbb{Z}^d \setminus \{\mathbf{0}\} \\ |h_j| < \mathcal{L} x^{\delta_j}}} \Bigg| \sum_{x < m \leqslant ax} f(m) \mathbf{e}\left(h_1 m^{\gamma_1} + \cdots + h_d m^{\gamma_d}\right) \Bigg| \Bigg) \Bigg),$$

where $\boldsymbol{h} = (h_1, \cdots, h_d)$.

Proof. See Proposition 6.2 in [12]. □

2.3. Exponential sum estimate

The following lemmas are the key to Theorem 1.1. We first define the exponential sum

$$\mathcal{S} = \sum_{\substack{0 < |h_j| < x^{\delta_j} \mathcal{L} \\ j = 1, \cdots, d}} \Bigg| \sum_{\substack{m \sim M \\ mn \sim x}} \sum_{n \sim N} a_m b_n \mathbf{e}\left(h_1 (mn)^{\gamma_1} + \cdots + h_d (mn)^{\gamma_d}\right) \Bigg|,$$

where $\mathcal{L} = \log x$, $N \geqslant 1, M \geqslant 1, MN \asymp x$, $|a_m| \ll x^{\varepsilon}$ and $|b_n| \ll x^{\varepsilon}$ for any $\varepsilon > 0$. If $b_n = 1$ or $b_n = \log n$, we call it a Type I sum and denote it as $\mathcal{S}_I$; otherwise we call it as a Type II sum and denote it as $\mathcal{S}_{II}$. We first have the following two lemmas.

Lemma 2.2. *Assume that $\varepsilon > 0$ is a sufficiently small real number, $\sigma < 1/(d+1)$ and $M < x^{29/42 - 2\sigma - \varepsilon}$. Then we have*

$$\mathcal{S}_I \ll x^{1-\varepsilon}.$$

Proof. See Proposition 7.1 in [12]. □

Lemma 2.3. *Assume that $\varepsilon > 0$ is a sufficiently small real number, $\sigma < 1/(d+1)$ and $x^{117\sigma/20 + \varepsilon} < M < x^{1 - 2\sigma - \varepsilon}$. Then we have*

$$\mathcal{S}_{II} \ll x^{1-\varepsilon}.$$

Proof. See Proposition 7.2 in [12]. □

Using a decomposition of k-fold divisor function and the above two lemmas, we have the following lemma.

Lemma 2.4. *Let γ_i, δ_i, $i = 1, \cdots, d$ be defined in Theorem 1.1. Define*

$$S = \sum_{\substack{\boldsymbol{h}\in\mathbb{Z}^d\setminus\{0\}\\ |h_i|<\mathcal{L}x^{\delta_i}}} \left|\sum_{n\sim x} d_k(n)\mathbf{e}(h_1n^{\gamma_1} + \cdots + h_dn^{\gamma_d})\right|.$$

Assume that σ be defined in Theorem 1.1 satisfying the following conditions

- *If $2 \leqslant k \leqslant 7$,*

$$\sigma < \min\left(\frac{290}{3297}, \frac{1}{2k}, \frac{1}{d+1}\right).$$

- *If $k \geqslant 8$,*

$$\sigma < \min\left(\frac{290}{4137}, \frac{1}{d+1}\right).$$

Then, we have the bound

$$S \ll x^{1-\varepsilon}.$$

Proof. The proof proceeds by analyzing the inner sum. We apply the definition

$$d_k(n) = \sum_{n=m_1\ldots m_k} 1.$$

We then apply a dyadic splitting argument to the variables $m_1, \ldots, m_k$. This decomposes S of the form

$$S = \sum_{\substack{\boldsymbol{h}\in\mathbb{Z}^d\setminus\{0\}\\ |h_i|<\mathcal{L}x^{\delta_i}}} \left|\sum_{m_1\sim M_1} \cdots \sum_{m_k\sim M_k} \mathbf{e}\left(\sum_{i=1}^{d} h_i(m_1\ldots m_k)^{\gamma_i}\right)\right|,$$

where $M_1 \cdots M_k \asymp x$. Without loss of generality, we assume $M_1 \leqslant \cdots \leqslant M_k$, which implies $M_k \geqslant x^{1/k}$. We will show that S can always be bounded by Lemma 2.2 and Lemma 2.3. We now consider the following three cases based on the value of k and M_k.

Case I: When $M_k \geqslant x^{13/42+2\sigma+\varepsilon}$, we define

$$N = M_k, \quad M = M_1 \cdots M_{k-1}, \quad n = m_k \quad \text{and} \quad m = m_1 \cdots m_{k-1}.$$

In this case we find that

$$S = \sum_{\substack{\boldsymbol{h}\in\mathbb{Z}^d\setminus\{0\}\\ |h_i|<\mathcal{L}x^{\delta_i}}} \left|\sum_{m\sim M}\sum_{n\sim N} a_mb_n\mathbf{e}(h_1(mn)^{\gamma_1} + \cdots + h_d(mn)^{\gamma_d})\right|,$$

where $b_n = 1$ and

$$a_m = \sum_{m_1 \cdots m_{k-1} = m} 1 \ll x^{\varepsilon}.$$

Hence in this case S is a sum of Type I and $M < x^{29/42-2\sigma-\varepsilon}$. From Lemma 2.2, we have

$$S \ll x^{1-\varepsilon}$$

provided that $\sigma < 1/(d+1)$.

Case II: For $2 \leqslant k \leqslant 7$ and $x^{1/k} \leqslant M_k < x^{13/42+2\sigma+\varepsilon}$, we define

$$N = M_k, \quad M = M_1 \cdots M_{k-1}, \quad n = m_k \quad \text{and} \quad m = m_1 \cdots m_{k-1}.$$

Hence S is still a sum of Type I and moreover is also a sum of Type II by the definition. Note that

$$x^{29/42-2\sigma-\varepsilon} < M \leqslant x^{1-1/k}.$$

To make our Type I and Type II sum estimate useful, we have that

$$\begin{cases} 1 - \frac{1}{k} < 1 - 2\sigma, \\ \frac{117\sigma}{20} < \frac{29}{42} - 2\sigma. \end{cases}$$

It follows that

$$\sigma < \min\Big(\frac{290}{3297}, \frac{1}{2k}\Big).$$

Hence from Lemma 2.2 and Lemma 2.3, we have

$$S \ll x^{1-\varepsilon}$$

provided that

$$\sigma < \min\Big(\frac{290}{3297}, \frac{1}{2k}, \frac{1}{d+1}\Big).$$

Case III: For $k \geqslant 8$ and $x^{1/k} \leqslant M_k < x^{13/42+2\sigma+\varepsilon}$, it implies that $M_i < x^{13/42+2\sigma+\varepsilon}$ holds for all $i = 1, 2, \cdots, k-1$. Suppose that l is the first positive integer satisfying the condition $x^{2\sigma}$, which gives that $l \geqslant 2$. Thus

$$x^{2\sigma} \leqslant M_1 \cdots M_l = M_1 \cdots M_{l-1} M_l \leqslant x^{2\sigma} x^{13/42+2\sigma+\varepsilon} = x^{13/42+4\sigma+\varepsilon}.$$

Define

$$N = M_1 \cdots M_l, \quad M = M_{1+1} \cdots M_k, \quad n = m_1 \cdots m_l \quad \text{and} \quad m = m_{1+1} \cdots m_k.$$

Thus S is a Type II sum of the form

$$S=\sum_{\substack{\boldsymbol{h}\in\mathbb{Z}^d\setminus\{0\}\\ |h_i|<\mathcal{L}x^{\delta_i}}}\left|\sum_{m\sim M}\sum_{n\sim N}a_m b_n\mathbf{e}(h_1(mn)^{\gamma_1}+\cdots+h_d(mn)^{\gamma_d})\right|,$$

where

$$a_m=\sum_{m_l\cdots m_k=m}1\ll x^{\varepsilon}\quad\text{and}\quad b_n=\sum_{m_1\cdots m_l=n}1\ll x^{\varepsilon}.$$

Thus from Lemma 2.3, we have

$$S\ll x^{1-\varepsilon}$$

if

$$\frac{117\sigma}{20}<\frac{29}{42}-4\sigma\quad\text{and}\quad\sigma<\frac{1}{d+1}.$$

It follows that

$$\sigma<\min\Big(\frac{290}{4137},\frac{1}{d+1}\Big).$$

Hence we finish the proof of this key lemma. □

3. Proof of theorem 1.1

Define $\mathcal{L}=\log x$. By Lemma 2.1, for $1<a<1+\mathcal{L}^{-1}$ we have that

$$\begin{aligned}\sum_{\substack{x<n\leqslant ax\\ n\in\mathcal{N}^{\boldsymbol{c}}}}d_k(n)&=\sum_{x<n\leqslant ax}d_k(n)\mathbf{1}_{c_1}(n)\cdots\mathbf{1}_{c_d}(n)\\ &=\gamma_1\cdots\gamma_{\rm d}x^{-\sigma}\Bigg((1+O(\mathcal{L}^{-1}))\sum_{x<n\leqslant ax}d_k(n)\\ &\quad+O\Bigg(\sum_{\substack{\boldsymbol{h}\in\mathbb{Z}^d\setminus\{\boldsymbol{0}\}\\ |h_j|<\mathcal{L}x^{\delta_j}}}\left|\sum_{x<n\leqslant ax}d_k(n)\mathbf{e}\left(h_1n^{\gamma_1}+\cdots+h_dn^{\gamma_d}\right)\right|\Bigg)\Bigg).\end{aligned}\tag{3.1}$$

Take $k=\left\lfloor\frac{\log x}{\log a}\right\rfloor+1$ to make sure that

$$a^{-k}x<1.$$

Hence we deduce that

$$\sum_{\substack{n\leqslant x\\ n\in\mathcal{N}^{\boldsymbol{c}}}}d_k(n)=\sum_{j=1}^{k}\sum_{\substack{a^{-j}x<n\leqslant a^{-j+1}x\\ n\in\mathcal{N}^{(\boldsymbol{c})}}}d_k(n).$$

By (3.1), we arrive at

$$\begin{aligned}\sum_{\substack{n\leqslant x\\ n\in\mathcal{N}^{\mathbf{c}}}} d_k(n) &= \sum_{j=1}^{k}\Bigg(\gamma_1\cdots\gamma_d(a^{-j}x)^{-\sigma}\Bigg((1+O(\mathcal{L}^{-1}))\sum_{a^{-j}x<n\leqslant a^{-j+1}x} d_k(n)\\ &\quad + O\Bigg(\sum_{\substack{\boldsymbol{h}\in\mathbb{Z}^d\setminus\{\mathbf{0}\}\\ |h_j|<\mathcal{L}(a^{-j}x)^{\delta_j}}}\Bigg|\sum_{a^{-j}x<n\leqslant a^{-j+1}x} d_k(n)\mathbf{e}\left(h_1n^{\gamma_1}+\cdots+h_dn^{\gamma_d}\right)\Bigg|\Bigg)\Bigg)\Bigg)\\ &= S_1+O(S_2),\end{aligned}$$

where

$$S_1=\gamma_1\cdots\gamma_d(1+O(\mathcal{L}^{-1}))\sum_{j=0}^{k}\sum_{a^{-j}x<n\leqslant a^{-j+1}x}(a^{-j}x)^{-\sigma}d_k(n)$$

and

$$S_2=\sum_{j=1}^{k}(a^{-j}x)^{-\sigma}\sum_{\substack{\boldsymbol{h}\in\mathbb{Z}^d\setminus\{\mathbf{0}\}\\ |h_j|<\mathcal{L}(a^{-j}x)^{\delta_j}}}\Bigg|\sum_{a^{-j}x<n\leqslant a^{-j+1}x} d_k(n)\mathbf{e}\left(h_1n^{\gamma_1}+\cdots+h_dn^{\gamma_d}\right)\Bigg|.$$

With standard methods, it follows that

$$\begin{aligned}S_1&=\gamma_1\cdots\gamma_d(1+O(\mathcal{L}^{-1}))\sum_{j=1}^{k}\sum_{a^{-j}x<n\leqslant a^{-j+1}x}\big(n^{-\sigma}+(a^{-j}x)^{-\sigma}-n^{-\sigma}\big)d_k(n)\\ &=\gamma_1\cdots\gamma_d(1+O(\mathcal{L}^{-1}))\sum_{n\leqslant x}d_k(n)n^{-\sigma}+O\Bigg(\sum_{j=1}^{k}\sum_{a^{-j}x<n\leqslant a^{-j+1}x}\big((a^{-j}x)^{-\sigma}-n^{-\sigma}\big)d_k(n)\Bigg).\end{aligned}$$

Note that for $a^{-j}x<n\leqslant a^{-j+1}x$,

$$(a^{-j}x)^{-\sigma}-n^{-\sigma}\ll(a^{-j}x)^{-\sigma}-(a^{-j+1}x)^{-\sigma}\ll(a^{-j}x)^{-\sigma}(1-a^{-\sigma})\ll\mathcal{L}^{-1}(a^{-j}x)^{-\sigma}.$$

Hence we have that

$$\begin{aligned}S_1&=(\gamma_1\cdots\gamma_d)(1+O(\mathcal{L}^{-1}))\sum_{n\leqslant x}d_k(n)n^{-\sigma}\\ &=(\gamma_1\ldots\gamma_d)(1+O((\log x)^{-1}))x^{1-\sigma}F_{k-1}(\log x)\end{aligned}$$

where $F_{k-1}(\log x)$ is a polynomial of degree $k-1$, which can be calculated as

$$F_{k-1}(\log x)=\operatorname*{Res}_{s=1-\sigma}x^{s-1}\zeta^k(s+\sigma)s^{-1}$$

by a similar calculation in [13], Chapter 13.

Next we turn to estimate S_2. By lemma 2.4, under the conditions of σ, we have

$$\sum_{\substack{\boldsymbol{h}\in\mathbb{Z}^d\setminus\{\mathbf{0}\}\\ |h_j|<\mathcal{L}(a^{-j}x)^{\delta_j}}}\Bigg|\sum_{a^{-j}x<n\leqslant a^{-j+1}x} d_k(n)\mathbf{e}\left(h_1n^{\gamma_1}+\cdots+h_dn^{\gamma_d}\right)\Bigg|\ll(a^{-j}x)^{1-\varepsilon}.$$

Hence we deduce that

$$S_2 \ll \sum_{j=1}^{k} (a^{-j}x)^{1-\sigma-\varepsilon}.$$

Recall that $k = \left\lfloor \frac{\log x}{\log a} \right\rfloor + 1$, it follows that

$$S_2 \ll x^{1-\sigma-\varepsilon}.$$

Hence we finish the proof of Theorem 1.1.

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Author contribution statement

All authors have accepted responsibility for the entire content of this manuscript and consented to its submission to the journal, reviewed all the results and approved the final version of the manuscript. All authors wrote the manuscript and are considered to have equal contributions.

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