Informatique théorique et applications

A. Lozano

Bounded queries to arbitrary sets

Informatique théorique et applications, tome 30, nº 2 (1996), p. 91-100.

<http://www.numdam.org/item?id=ITA_1996__30_2_91_0>

Informatique théorique et Applications/Theoretical Informatics and Applications
(vol. 30, n° 2, 1996, pp. 91-100)

BOUNDED QUERIES TO ARBITRARY SETS

by A. LOZANO [1]

Abstract. – *We prove that if* $\mathrm{P}^{A}_{k\text{-}T} = \mathrm{P}^{A}_{(k+1)\text{-}T}$ *for some* k *and an arbitrary set* A, *then* A *is reducible to its complement under a relativized nondeterministic conjunctive reduction. By substituting* A *by different sets, we derive some known facts such as Kadin's theorem [13] and its extension to the class* $\mathrm{C_{=}P}$ *[5, 8].*

1. INTRODUCTION

We are interested in the hierarchy of sets accepted by machines that make a bounded number of queries to an arbitrary fixed set. Normally, only sets from some well-known complexity class, such as NP, have been considered as oracles for such hierarchies, and the results derived have increased our knowledge about the relationships among these classes or the hierarchies related to them (such as the boolean or the polynomial-time hierarchies). Here we consider arbitrary oracles in an attempt to generalize known results and provide a basis for further developments.

For any set A, we call $\mathrm{P}^{A}_{k\text{-}T}$ the class of sets computable by deterministic polynomial-time machines that make k queries to oracle A. If we require the queries to be made in parallel, we denote the resulting class by $\mathrm{P}^{A}_{k\text{-}tt}$. Note that in order to decide sets in $\mathrm{P}^{A}_{k\text{-}T}$, a machine can make a query to A that depends on the answers to previous queries (they are called serial or adaptive queries) while in the case of $\mathrm{P}^{A}_{k\text{-}tt}$ the queries depend exclusively on the input (parallel or non-adaptive queries).

[1] Departament L.S.I., Pau Gargallo 5, E08028-Barcelona, Catalonia, (email: lozano@lsi.upc.es).
This research was partially supported by grant EE91/2–298 of the *Comissió Interdepartamental de Recerca i Innovació Tecnològica* and by the ESPRIT-II Basic Research Actions Program of the European Community under contract No. 3075 (project ALCOM).

0988-3754/96/02/$ 4.00/© AFCET-Gauthier-Villars

The hierarchy

$$\mathrm{P}^A_{1\text{-}T}, \mathrm{P}^A_{2\text{-}T}, \mathrm{P}^A_{3\text{-}T}, \ldots$$

is called the bounded-query hierarchy relative to A, while its parallel counterpart is called the bounded parallel query hierarchy relative to A [4]. Sometimes we will consider the bounded query hierarchies relative to classes, instead of sets; they are defined in the obvious way: for a class of sets $\mathcal{C}$, $\mathrm{P}^{\mathcal{C}}_{k\text{-}T}$ is $\bigcup_{A \in \mathcal{C}} \mathrm{P}^A_{k\text{-}T}$. We will use the property that if A is a $\leq^{\mathrm{P}}_m$-complete set for $\mathcal{C}$, then $\mathrm{P}^{\mathcal{C}}_{k\text{-}T} = \mathrm{P}^A_{k\text{-}T}$.

Some properties about sets whose bounded query hierarchies collapse are known when these sets are taken from a uniform class. The results of Kadin [13, 14] ($\mathrm{P}^{\mathrm{NP}}_{k\text{-}T} = \mathrm{P}^{\mathrm{NP}}_{(k+1)\text{-}T}$ implies $\mathrm{PH} = \Delta^p_3$), Chang [6] ($A \in \mathrm{NP}$ and $\mathrm{P}^A_{k\text{-}T} = \mathrm{P}^A_{(k+1)\text{-}T}$ imply $A \in \widehat{\mathrm{low}_3}$), Beigel, Chang, and Ogihara [5], and Green [8] ($\mathrm{P}^{\mathrm{C_=P}}_{k\text{-}T} = \mathrm{P}^{\mathrm{C_=P}}_{(k+1)\text{-}T}$ implies $\mathrm{PH}^{\mathrm{PP}} = \mathrm{P}^{\mathrm{NP}^{\mathrm{PP}}}$) can all be seen as consequences of the collapse of different bounded query hierarchies. However, no properties of an arbitrary set A satisfying the simple equation $\mathrm{P}^A_{k\text{-}T} = \mathrm{P}^A_{(k+1)\text{-}T}$, for some k, were known. We prove in this paper that, in this case, A is reducible to its complement under a weak reducibility.

In order to state the main result more precisely, consider the following nondeterministic reducibilities, defined in [15]. We say that a set A is $\leq^{\mathrm{NP}}_m$-reducible to a set B if there is a nondeterministic polynomial-time machine that, on input x, generates a word belonging to B, for some computation path, if and only if $x \in A$. Similarly, we say that A is $\leq^{\mathrm{NP}}_{ctt}$-reducible to B if there is a nondeterministic polynomial-time machine that, on input x, generates a tuple of words that belong to B, for some computation path, if and only if $x \in A$. For any reducibility $\leq^{\mathrm{NP}}_r$, we write $\leq^{\mathrm{NP},C}_r$ if the machine defining the reducibility has unrestricted access to oracle C.

In Section 3 we prove that if the bounded-query hierarchy relative to A collapses, then there exists a sparse set S such that $A \leq^{\mathrm{NP},S}_{ctt} \overline{A}$. In Section 4 we show that the previously known consequences of the collapse of some bounded query hierarchies can be proved from our general theorem, in particular Kadin's theorem [13] and a similar result for the class $\mathrm{C_=P}$ [5, 8]. It should be mentioned here that Kadin's theorem was improved by Chang and Kadin [7] with a collapse of the polynomial hierarchy to $\mathrm{P}^{\Sigma^p_2}_{(k-1)-tt}$.

2. PRELIMINARIES

We assume that the reader is familiar with the basic structural complexity notions, and in particular with classes such as P, NP, and PH. The main result in this paper will be applied to the classes NP and $C_=P$. Let $\Sigma = \{0,1\}$, and define the class $C_=P$ as follows:

DEFINITION 2.1 [19, 21]: *A set A is in $C_=P$ if there exists a polynomial time nondeterministic Turing machine M and a polynomial time computable function f such that for every $x \in \Sigma^*$, it holds that $x \in A$ if and only if the number of accepting computation paths of M on input x is equal to $f(x)$.*

We denote with $C_{\neq}P$ the class of sets whose complements belong to $C_=P$. Also, the notation NE stands for the class $\bigcup_c \mathrm{NTIME}[2^{cn}]$, and NEXP for $\bigcup_k \mathrm{NTIME}[2^{n^k}]$. Following [6], we construct "boolean languages" with respect to a set.

DEFINITION 2.2: *For any set A, we define the following sets:*

$$\begin{aligned}
\mathrm{BL}_A(1) &= A\\
\mathrm{BL}_A(2k) &= \{(x_1,\ldots,x_{2k-1},x_{2k}) :\\
&\qquad (x_1,\ldots,x_{2k-1}) \in \mathrm{BL}_A(2k-1) \textit{ and } x_{2k} \in \overline{A}\}\\
\mathrm{BL}_A(2k+1) &= \{(x_1,\ldots,x_{2k},x_{2k+1}) :\\
&\qquad (x_1,\ldots,x_{2k}) \in \mathrm{BL}_A(2k) \textit{ or } x_{2k+1} \in A\}\\
\text{co-}\mathrm{BL}_A(1) &= \overline{A}\\
\text{co-}\mathrm{BL}_A(2k) &= \{(x_1,\ldots,x_{2k-1},x_{2k}) :\\
&\qquad (x_1,\ldots,x_{2k-1}) \in \text{co-}\mathrm{BL}_A(2k-1) \textit{ or } x_{2k} \in A\}\\
\text{co-}\mathrm{BL}_A(2k+1) &= \{(x_1,\ldots,x_{2k},x_{2k+1}) :\\
&\qquad (x_1,\ldots,x_{2k}) \in \text{co-}\mathrm{BL}_A(2k) \textit{ and } x_{2k+1} \in \overline{A}\}
\end{aligned}$$

We say that a sparse set S is *p-printable* if there is a polynomial-time oracle Turing machine which prints all the strings in S of a given length n on input 1^n. Other sets that are used in the proofs are defined next.

DEFINITION 2.3: *For any set A we define the following sets:*

$$\begin{aligned}
A^{\forall} &= \{x_1\#x_2\#\cdots\#x_n : n \geq 1 \textit{ and } (\forall i \leq n)[x_i \in A]\}\\
A^{\exists} &= \{(x_1,\ldots,x_n) : n \geq 1 \textit{ and } (\exists i \leq n)[x_i \in A]\}\\
\mathrm{PARITY}_k^A &= \{(x_1,\ldots,x_k) : \|\{i : 1 \leq i \leq k \textit{ and } x_i \in A\}\| \textit{ is odd}\}
\end{aligned}$$

To denote the set $\mathrm{PARITY}_k^{(A^\forall)}$, we will simply write $\mathrm{PARITY}_k^{A^\forall}$. In particular, in Claim 2 we will also consider the set $((A^\forall)^\forall)^\exists$, which equals $(A^\forall)^\exists$ since it trivially holds that $(A^\forall)^\forall = A^\forall$. This last fact explains the special notation used to encode the instances of $A^\forall$.

We define now the relativized nondeterministic reducibilities $\leq_m^{\mathrm{NP}}$ and $\leq_{ctt}^{\mathrm{NP}}$ (*see* [15, 1] for the nonrelativized versions).

Definition 2.4: *For any three sets A, B and C, we say that*

(1) A is polynomial-time nondeterministic many-one reducible to B relative to C (denoted $A \leq_m^{\mathrm{NP},C} B$*), if there exists a polynomial-time nondeterministic oracle Turing transducer M such that for every* $x \in \Sigma^*$,

(a) for each computation path of M on input x and oracle C, M outputs some string, and

(b) $x \in A$ *if and only if there exists some computation path of M on input x and oracle C that outputs some string* $y \in B$.

(2) A is polynomial-time nondeterministic conjunctive truth-table reducible to B relative to C (denoted $A \leq_{ctt}^{\mathrm{NP},C} B$*), if there exists a polynomial-time nondeterministic oracle Turing transducer M such that for every* $x \in \Sigma^*$,

(a) for each computation path of M on input x and oracle C, M outputs a string of the form $y_1\#\dots\#y_k$, *and*

(b) $x \in A$ *if and only if there exists some computation path of M on input x and oracle C that outputs some string* $y_1\#\dots\#y_k$ *such that* $\{y_1,\dots,y_k\} \subseteq B$.

Definition 2.5: *For any set B, we define the following classes related to the above nondeterministic reducibilities:*

$$\mathrm{NP}_m^B = \{A : A \leq_m^{\mathrm{NP}} B\}$$
$$\mathrm{NP}_{ctt}^B = \{A : A \leq_{ctt}^{\mathrm{NP}} B\}$$

Let us mention two easy observations involving the bounded-query hierarchies. The first observation is the *upward collapse property* of the $\mathrm{P}_{k\text{-}T}^A$ hierarchy. Whether it holds for the corresponding parallel hierarchy is not known.

Observation 2.6 [3]: *If* $\mathrm{P}_{k\text{-}T}^A = \mathrm{P}_{(k+1)\text{-}T}^A$, *then for all* $j > k$, $\mathrm{P}_{k\text{-}T}^A = \mathrm{P}_{j\text{-}T}^A$.

Observation 2.7 [3]: *For any set A, the* $\mathrm{P}_{k\text{-}T}^A$ *hierarchy collapses if and only if the* $\mathrm{P}_{k\text{-}tt}^A$ *hierarchy collapses.*

3. THE MAIN RESULT

In this section we show that if for some set A, there is a k such that $\mathrm{P}^{A}_{k\text{-}T} = \mathrm{P}^{A}_{(k+1)\text{-}T}$, then A is reducible to its complement under a weak reduction: $A \leq_{ctt}^{\mathrm{NP},S} \overline{A}$, for a certain sparse set S. After this, we will see how Kadin's theorem and its extension to the class $\mathrm{C}_{=}\mathrm{P}$ can be easily derived from our result.

The next result derives directly from a technical lemma of Chang [6].

LEMMA 3.1: *If* $\mathrm{BL}_A(k) \leq_m^{\mathrm{P}} \text{co-}\mathrm{BL}_B(k)$ *for some* k, *then* $\overline{A} \leq_m^{\mathrm{NP},S} B$, *where* S *is a p-printable set in* $\mathrm{P}^{\mathrm{NP}^{\mathrm{NP}^{B}_{m}}}$.

Proof: The same proof as that of Lemma 1 in [6] can be used to prove this lemma. The difference is that we take two arbitrary sets A and B (not necessarily from NP as in [6]), and then the final NP^S algorithm that decides $\overline{A}$ must be considered in this case as the machine that witnesses the reduction $\overline{A} \leq_m^{\mathrm{NP},S} B$. Also, the first algorithm in that proof shows that $S \in \Delta_3^p$ but, as B is here an arbitrary set, it is not hard to check that $S \in \mathrm{P}^{NP^{\mathrm{NP}^{B}_{m}}}$. □

Now, we can prove the main theorem.

THEOREM 3.2: *For any set* A, *if* $\mathrm{P}^{A}_{k\text{-}T} = \mathrm{P}^{A}_{(k+1)\text{-}T}$ *for some* k, *then* $A \leq_{ctt}^{\mathrm{NP},S} \overline{A}$, *where* S *is a p-printable set in* $\mathrm{P}^{\mathrm{NP}^{\mathrm{NP}^{\overline{A}}_{ctt}}}$.

Proof: Let $j = 2^k - 1$. Therefore,

$$\begin{aligned}
\mathrm{P}^{A}_{2^j\text{-}tt} &\subseteq \mathrm{P}^{A}_{2^j\text{-}T} \\
&\subseteq \mathrm{P}^{A}_{k\text{-}T} && \text{by Observation 2.6} \\
&\subseteq \mathrm{P}^{A}_{(2^k-1)\text{-}tt} && (\textit{see } [4]) \\
&= \mathrm{P}^{A}_{j\text{-}tt}
\end{aligned}$$

Thus, the set $\mathrm{BL}_{\overline{A}}(2^j)$ is in $\mathrm{P}^{A}_{j\text{-}tt}$. Now, we claim that the set $\mathrm{PARITY}^{A^{\forall}}_{2^j}$ is $\leq_m^{\mathrm{P}}$-hard for $\mathrm{P}^{A}_{j\text{-}tt}$ and that $\mathrm{PARITY}^{A^{\forall}}_{2^j} \leq_m^{\mathrm{P}} \mathrm{BL}_{(A^{\forall})^{\exists}}(2^j)$. These two claims allow us to prove the following reductions:

$$\begin{aligned}
\mathrm{BL}_{\overline{A}}(2^j) &\leq_m^{\mathrm{P}} \mathrm{PARITY}^{A^{\forall}}_{2^j} && \text{by Claim 1} \\
&\leq_m^{\mathrm{P}} \mathrm{BL}_{(A^{\forall})^{\exists}}(2^j) && \text{by Claim 2} \\
&= \text{co-}\mathrm{BL}_{\overline{(A^{\forall})^{\exists}}}(2^j)
\end{aligned}$$

Thus, by Lemma 3.1, we have $A \leq_m^{\mathrm{NP};S} \overline{(A^\forall)^\exists}$ for a sparse set S which is p-printable in $\mathrm{P}^{\mathrm{NP}^{\mathrm{NP}_m^{\overline{(A^\forall)^\exists}}}}$. Observe that $\overline{(A^\forall)^\exists} \leq_m^{\mathrm{P}} (\overline{A}^\exists)^\forall \leq_m^{\mathrm{NP}} \overline{A}^\forall \leq_{ctt}^{\mathrm{P}} \overline{A}$. Therefore, $A \leq_{ctt}^{\mathrm{NP}.S} \overline{A}$, where S is a p-printable set in $\mathrm{P}^{\mathrm{NP}^{\mathrm{NP}_{ctt}^{\overline{A}}}}$.

Next, we prove the claims.

Claim 1: *For any n, $\mathrm{PARITY}_{2^n}^{A^\forall}$ is $\leq_{\mathrm{m}}^{\mathrm{p}}$-hard for $\mathrm{P}_{n\text{-}tt}^{A}$.*

Proof: Let $L \in \mathrm{P}_{n\text{-}tt}^{A}$, and let M be a machine that witnesses it. Let $q_i(x)$ be the i-th query made by M on input x.

Given an instance x, it is easy to compute a boolean formula f_x in polynomial time, such that:

$$x \in L \iff f_x(\chi_A(q_1(x)), \ldots, \chi_A(q_n(x))) = 1.$$

By standard methods, f_x can be transformed into a polynomial p_x over $Z/2$ where, as usual, 0 denotes false and 1 denotes true. Now, transform p_x into a boolean formula g_x that has conjunctions for every monomial and parity operators for sums. This is a big parity of at most 2^n terms formed by conjunctions. Now, we have

$$x \in L \iff g_x(\chi_A(q_1(x)), \ldots, \chi_A(q_n(x))) = 1.$$

where g_x can easily be transformed into an instance of $\mathrm{PARITY}_{2^n}^{A^\forall}$ in the obvious way. For example, formula $\chi_A(q_1(x)) \oplus (\chi_A(q_1(x)) \wedge \chi_A(q_2(x)))$ would be transformed into the instance $(q_1(x), q_1(x)\#q_2(x))$. Therefore, x is in A if and only if the instance we have constructed belongs to $\mathrm{PARITY}_{2^n}^{A^\forall}$. $\square$

Claim 2: *For any n, $\mathrm{PARITY}_n^{A^\forall} \leq_{\mathrm{m}}^{\mathrm{p}} \mathrm{BL}_{(A^\forall)^\exists}(n)$.*

Proof: First we reduce PARITY_n^{A} to $\mathrm{BL}_{(A^\forall)^\exists}(n)$. Let $x = (x_1, \ldots, x_n)$ be an instance of PARITY_n^{A}. We can check whether at least i inputs belong to A by trying each possible combination of i inputs, which yields an $(A^\forall)^\exists$ predicate. Let $f_i(x)$ be this predicate. Now, an odd number of the inputs belong to A if and only if the predicate

$$f_1(x) \in (A^\exists)^\forall \wedge \neg(f_2(x) \in (A^\forall)^\exists \wedge \neg(f_3(x) \in (A^\forall)^\exists \wedge \cdots))$$

is true, this is, if $f(x) = \langle f_1(x), \langle f_2(x), \ldots, f_n(x)\rangle\rangle$ belongs to $\mathrm{BL}_{(A^\forall)^\exists}(n)$.

By the comment after Definition 2.3, if we simply substitute A by $A^{\forall}$ in the above proof, we get a reduction from $\mathrm{PARITY}_n^{A^{\forall}}$ to $\mathrm{BL}_{(A^{\forall})^{\exists}}(n)$, as desired. □

□ **Proof of Theorem 3.2.**

4. SOME CONSEQUENCES: THE BOUNDED QUERY HIERARCHIES OVER NP AND $\mathrm{C}_=\mathrm{P}$

The most interesting applications of Theorem 3.2 can be found for classes which are closed under $\leq_{ctt}^{\mathrm{NP}}$-reducibility, a property satisfied by any class that is closed under the $\leq_m^{\mathrm{NP}}$ and $\leq_{ctt}^{\mathrm{P}}$-reducibilities. Three classes that satisfy these conditions are NP, $\mathrm{C}_{\neq}\mathrm{P}$ and NEXP [1]. We show now the consequences for the first two classes (for NE, read the comment in the next section).

Now, we show that if we take the set A in our main result from the class of NP-complete sets (for $\leq_m^{\mathrm{P}}$-reducibility), we obtain Kadin's theorem.

COROLLARY 4.1 [13]: *If* $\mathrm{P}_{k\text{-}T}^{\mathrm{NP}} = \mathrm{P}_{(k+1)\text{-}T}^{\mathrm{NP}}$ *for some k, then* $\mathrm{PH} \subseteq \Delta_3^p$.

Proof: The hypothesis is equivalent to the equality $\mathrm{P}_{k\text{-}T}^{\mathrm{SAT}} = \mathrm{P}_{(k+1)\text{-}T}^{\mathrm{SAT}}$, which is also equivalent to $\mathrm{P}_{k\text{-}T}^{\overline{\mathrm{SAT}}} = \mathrm{P}_{(k+1)\text{-}T}^{\overline{\mathrm{SAT}}}$. Theorem 3.2 implies that $\overline{\mathrm{SAT}} \leq_{ctt}^{\mathrm{NP}.S} \mathrm{SAT}$ for some p-printable set S in $\mathrm{P}^{\mathrm{NP}^{\mathrm{NP}_{ctt}^{\mathrm{SAT}}}}$. As NP is closed under $\leq_{ctt}^{\mathrm{NP}}$-reducibility, this means that co-NP $\subseteq \mathrm{NP}^S$ for some p-printable set S in Δ_3^p, which implies that $\mathrm{PH} \subseteq \Delta_3^p$, by the results in [11]. □

The next result was proved by Green [8] and independently, in a stronger form, by Beigel, Chang, and Ogihara [5]. We present it here also as a consequence of our main result.

COROLLARY 4.2 [8, 5]: *If* $\mathrm{P}_{k\text{-}T}^{\mathrm{C}_=\mathrm{P}} = \mathrm{P}_{(k+1)\text{-}T}^{\mathrm{C}_=\mathrm{P}}$ *for some k, then* $\mathrm{PH}^{\mathrm{PP}} \subseteq \mathrm{P}^{\mathrm{NP}^{\mathrm{PP}}}$.

Proof: By a similar argument to that of the previous corollary, we can state that $\mathrm{C}_=\mathrm{P} \subseteq \mathrm{C}_{\neq}\mathrm{P}^S$, for some p-printable set S in $\mathrm{P}^{\mathrm{NP}^{\mathrm{C}_{\neq}\mathrm{P}}}$, since $\mathrm{C}_{\neq}\mathrm{P}$ is closed under $\leq_{ctt}^{\mathrm{NP}}$-reducibility [1].

Consider the inclusions $\mathrm{NP}^{\mathrm{C}_{\neq}\mathrm{P}^S} \subseteq \exists\mathrm{C}_{\neq}\mathrm{P}^S \subseteq \mathrm{C}_{\neq}\mathrm{P}^S$, where the first one derives from Torán [20], and the second one from the fact that $\mathrm{C}_{\neq}\mathrm{P}$ is closed under $\leq_m^{\mathrm{NP}}$-reducibility. By induction we have that $\mathrm{PH}^{\mathrm{C}_{\neq}\mathrm{P}^S} \subseteq \mathrm{C}_{\neq}\mathrm{P}^S$.

[1] The case of NP is well known. The closure of $\mathrm{C}_{\neq}\mathrm{P}$ under the $\leq_m^{\mathrm{NP}}$ and $\leq_{ctt}^{\mathrm{NP}}$-reducibilities is proved in [5] and esentially the same results can be found in [9]. For NE, the proof is similar to that of NP. For related results. *see* [17].

Now, also from [20] we know that $\mathrm{PH}^{\mathrm{PP}} \subseteq \mathrm{PH}^{\mathrm{C_{=}P}}$, and then $\mathrm{PH}^{\mathrm{PP}} \subseteq \mathrm{PH}^{\mathrm{C_{\neq}P}^S} \subseteq \mathrm{C_{\neq}P}^S$. But this last class can be shown to be included in $\mathrm{P}^{\mathrm{NP}^{\mathrm{C_{\neq}P}}}$, by using the information that we have about S, and hence in $\mathrm{P}^{\mathrm{NP}^{\mathrm{PP}}}$. Therefore $\mathrm{PH}^{\mathrm{PP}} \subseteq \mathrm{P}^{\mathrm{NP}^{\mathrm{PP}}}$. □

5. CONCLUSIONS AND OPEN PROBLEMS

We have given the first known property for an arbitrary set A when it satisfies $\mathrm{P}^A_{k\text{-}T} = \mathrm{P}^A_{(k+1)\text{-}T}$ for some k. We have proved that, in this case, A is reducible to its complement under a weak reduction: $A \leq^{\mathrm{NP};S}_{ctt} \overline{A}$, for some sparse set S (whose complexity is specified in terms of that of A). We have also seen that this leads us to conclude some known facts about the collapse of the bounded query hierarchies relative to NP and $\mathrm{C_{=}P}$ (or about the collapse of their respective boolean hierarchies, which is equivalent in these cases). This generalizes the mentioned results for the classes NP and $\mathrm{C_{=}P}$, but it is not quite as sharp as the best collapse known in the case of NP [7].

A first question left open in this paper is: Can we conclude some interesting consequence for some other complexity class? It seems interesting to apply our theorem to the class of nondeterministic exponential time since, although Hemachandra [10] proved the collapse of the strong exponential hierarchy to P^{NE}, it is not known whether a stronger collapse (as $\mathrm{P}^{\mathrm{NE}}_{k\text{-}T} = \mathrm{P}^{\mathrm{NE}}_{(k+1)\text{-}T}$) could cause some unlikely consequence. In fact, the main result in this paper implies that if the bounded-query hierarchy over NE collapses, then co-NEXP $\subseteq$ NEXP/poly [2]. Does this imply that NEXP = co-NEXP?

Apparently more difficult problems arise when one tries to obtain some consequence from the equality of the classes $\mathrm{P}^A_{\log\text{-}T}$ and P^A. Note that, however, we know some consequences of the facts [3] $\mathrm{PF}^{A[\log]} = \mathrm{PF}^A$, $\mathrm{PF}^{A[k]} = \mathrm{PF}^{A[k+1]}$, and now $\mathrm{P}^A_{k\text{-}T} = \mathrm{P}^A_{(k+1)\text{-}T}$. This seems to be a hard question since, by taking A = SAT, this would give us a consequence of the fact $\Delta^p_2 = \Theta^p_2$, which is still unknown.

[2] Note that it is easy to prove by padding arguments that the bounded-query hierarchy over NE collapses if and only if the bounded-query hierarchy over NEXP collapses.

[3] For the first one, from the results by Amir, Beigel, and Gasarch [2] it follows that $\mathrm{PF}^{A[\log]} = \mathrm{PF}^A$ implies that $A \in \mathrm{NP}/poly \cap$ co-NP/poly. The second fact is equivalent to saying that A is cheatable; also in [2] it is shown that in this case $A \in$ P/poly.

ACKNOWLEDGEMENTS

The author would like to thank José Luis Balcázar, Richard Beigel, and Ricard Gavaldà for proofreading and the referees for their detailed and helpful comments.

REFERENCES

1. E. Allender, L. A. Hemachandra, M. Ogiwara, and O. Watanabe, Relating equivalence and reducibility to sparse sets, *SIAM Journal on Computing*, 1992, *2*, pp. 521-539.
2. A. Amir, R. Beigel, and W. I. Gasarch, Some connections between bounded query classes and non-uniform complexity, *Proceedings of the 5th Structure in Complexity Theory Conference*, 1990, pp. 232-243.
3. R. Beigel, Query-limited Reducibilities, *PhD thesis*, Stanford University, 1987.
4. R. Beigel, Bounded queries to SAT and the boolean hierarchy, *Theoretical Computer Science*, 1991, *84* (2), pp. 199-223.
5. R. Beigel, R. Chang, and M. Ogiwara, A relationship between difference hierarchies and relativized polynomial hierarchies, *Mathematical Systems Theory*, 1993, *26* (3), pp. 293-310.
6. R. Chang, On the structure of bounded queries to arbitrary NP sets, *SIAM Journal on Computing*, 1992, *21* (4), pp. 743-754.
7. R. Chang and J. Kadin, The boolean hierarchy and the polynomial hierarchy: a closer connection, *Proceedings of the 5th Structure in Complexity Theory Conference*, 1990, pp. 169-178. To appear in *SIAM Journal on Computing*.
8. F. Green, On the power of deterministic reductions to $C_{=}P$, Manuscript, April 1991.
9. T. Gunderman, N. Nasser, and G. Wechsung, A survey of counting classes, *Proceedings of the 5th Structure in Complexity Theory Conference*, 1990, pp. 140-153.
10. L. A. Hemachandra, Counting in structural complexity theory, *PhD thesis*, Cornell University, June 1987.
11. J. Kadin, Is one NP question as powerful as two?, *Technical Report TR 87-842*, Cornell University, June 1987.
12. J. Kadin, Restricted Turing reducibilities and the structure of the Polynomial Time Hierarchy, *PhD thesis*, Cornell University, February 1988.
13. J. Kadin, The polynomial time hierarchy collapses if the boolean hierarchy collapses, *SIAM Journal on Computing*, 1988, *17* (6), pp. 1263-1282.
14. J. Kadin, Erratum to [13], *SIAM Journal on Computing*, 1991, *20* (2), p. 104.
15. R. E. Ladner, N. A. Lynch, and A. L. Selman, A comparison of polynomial time reducibilities, *Theoretical Computer Science*, 1975, *1*, pp. 103-123.
16. M. Ogiwara, On the computational power of exact counting, unpublished manuscript, April 1990.
17. M. Ogiwara, Generalized theorems on relationships among reducibility notions to certain complexity classes, *Mathematical Systems Theory*, 1984, *27*, pp. 189-200.
18. M. Ogiwara and L. A. Hemachandra, A complexity theory for feasible closure properties, *Journal of Computer and System Sciences*, 1993, *46* (3), pp. 295-325.
19. J. Simon, On some central problems in computational complexity, *PhD thesis*, Cornell University, January 1975.

20. J. Torán, Structural properties of the Counting Hierarchy, *PhD thesis*, Universitat Politècnica de Catalunya, January 1990.
21. K. W. Wagner, The complexity of combinatorial problems with succinct input representation, *Acta Informatica*, *1986*, *23*, pp. 325-356.