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SOME CONSEQUENCES OF A RESULT OF EHRENFEUCHT AND ROZENBERG (*)

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Abstract. — *Recently, Ehrenfeucht and Rozenberg have proved that there are context-free languages that are not EDTOL languages. In the present note, some consequences of this result are pointed out with reference to certain open problems related to matrix languages.*

Résumé. — *Récemment, Ehrenfeucht et Rozenberg ont prouvé qu'il existe des langages indépendants du contexte qui ne sont pas des EDTOL-langages. Dans la présente note, nous indiquons quelques conséquences de ce résultat, en résolvant certains problèmes ouverts sur les langages matriciels.*

1. NOTATIONS

In order to save space we shall omit the definitions and we shall specify only the notations we use:

– $\mathscr{L}$ (EDTOL) is the family of languages generated by EDTOL systems (*see* [6]);

– $\mathscr{M}_f$ is the family of languages generated by matrix grammars of finite index [1];

– $\mathscr{S}\mathscr{M}$ is the family of simple matrix languages [7] and $\mathscr{S}\mathscr{M}_f$ is the family of finite index languages in $\mathscr{S}\mathscr{M}$;

– $\mathscr{L}_2$ is the family of context-free languages;

– $\mathrm{Ind}_m(L)$, $\mathrm{Ind}_{cf}(L)$ denote the index of a language L according to matrix grammars, respectively, to context-free grammars;

– D_i is the Dyck language over the vocabulary $\{a_1, \ldots, a_i, a'_1, \ldots, a'_i\}$, that is the language generated by the context-free grammar with the rules $S \to SS$, $S \to a_j\, S\, a'_j$, $j=1, 2, \ldots, i$, $S \to \lambda$ [16].

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2. OPEN PROBLEMS RELATED TO THE FAMILIES $\mathcal{M}_f$, $\mathcal{SM}$

The index of a matrix grammar was defined in [1] where it is proved that for any infinite language L in $\mathcal{M}_f$, the set of lengths of strings in L contains an infinite arithmetical progression (a similar result holds for languages in $\mathcal{L}_2$). In [1] it is asked (problem A) whether or not any context-free language has a finite index (clearly, according to matrix grammars).

The problem was partially solved by Salomaa [15] by proving that $\mathrm{Ind}_{cf}(D_1)$ is infinite. The existence of a context-free language of infinite index according to matrix grammars remained open.

In [3], Cremers, Mayer, Weiss claim that $\mathrm{Ind}_m(D_1)=2$, but, as in [8] it was pointed out, the proof in [3] is wrong; the finiteness of $\mathrm{Ind}_m(D_1)$ remained open. Note that if any Dyck language would be in $\mathcal{M}_f$, then $\mathcal{L}_2 \subset \mathcal{M}_f$ since the family $\mathcal{M}_f$ is a full-AFL [11] and any language L in $\mathcal{L}_2$ can be written as $h(D_i \cap R)$, where h is a homomorphism and R is a regular language [16].

In [7] Ibarra introduces the so-called simple matrix grammars, a class of grammars generating languages with many context-free-like properties. Similar properties (closure properties, decision results, pumping lemmas) were proved for the family $\mathcal{M}_f$ too (*see* [11, 12, 14]). The relation between the families $\mathcal{M}_f$ and $\mathcal{SM}$ was formulated as an open problem in various contexts [9, 11, 12]; in fact, we obviously have $\mathcal{M}_f - \mathcal{SM} = \emptyset$ since the language $\{a^n b^n c^n \mid n \geqq 1\}^*$ is in $\mathcal{M}_f$ but not in $\mathcal{SM}$ [7]. Therefore, the problem asks for a language in $\mathcal{SM}$ which has an infinite matrix index. A stronger formulation [12] asks for context-free languages which are not in $\mathcal{M}_f$ (problem related to the paper [3]).

In [9] we proved that $\mathcal{SM}_f \subset \mathcal{M}_f$ and one asks whether or not $\mathcal{SM}_f \subseteqq \mathcal{SM}$ is a proper inclusion. The problem was solved in [10] by proving that D_1 does not belong to $\mathcal{SM}_f$. In a similar way it follows that any D_i, $i \geqq 1$, is not in $\mathcal{SM}_f$.

3. THE SETTLEMENT OF THE ABOVE OPEN PROBLEMS

The result in [4] can be easily used to solve all the above open problems. Indeed, we have:

THEOREM: $\mathrm{Ind}_m(D_i)$ *is infinite for any* $i \geqq 2$.

Proof: In [2] one introduces the "parallel matrix grammars" as usual context-free matrix grammars with parallel derivation (each rule rewrites all occurrences of its left-hand side in the string to be rewritten). Let us denote by $\mathcal{PM}$ the family of languages generated by these grammars. In [2] it is proved that $\mathcal{M}_f \subset \mathcal{PM}$, strict inclusion.

In [13] it is proved that, in fact, we have $\mathcal{PM}=\mathcal{L}$ (EDTOL), therefore $\mathcal{M}_f \subset \mathcal{L}$ (EDTOL). Following [4], there are context-free languages which are not in $\mathcal{L}$ (EDTOL). Thus there are context-free languages which do not belong to $\mathcal{M}_f$ as well. Consequently, any context-free generator (any context-free language whose smallest AFL containing it equals the family of context-free languages) is not a matrix language of finite index. Any language D_i, $i \geqq 2$, is a context-free generator (*see* example 5.1.1, p. 139 [5]), hence $\mathrm{Ind}_m(D_i)$ is infinite for any $i \geqq 2$.

Consequently, (1) $\mathcal{L}_2$ and $\mathcal{M}_f$ are incomparable (clearly, $\mathcal{M}_f$ contains non-context-free languages), (2) $\mathcal{SM}$ and $\mathcal{M}_f$ are incomparable, (3) $\mathrm{Ind}_m(D_i)=\mathrm{Ind}_{cf}(D_i)=\infty$, $i \geqq 2$, and thus all the problems listed in section 2 are settled.

It remains open the problem to find context-free languages L such that $\mathrm{Ind}_m(L) < \mathrm{Ind}_{cf}(L)$ [or even $\mathrm{Ind}_m(L) < \mathrm{Ind}_{cf}(L)=\infty$]. Particularly, we do not know whether $\mathrm{Ind}_m(D_1)$ is finite or not.

Some problems settled by the result in [4] were considered in [4] too. Perhaps there are other problems which can be solved using the same result.

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